

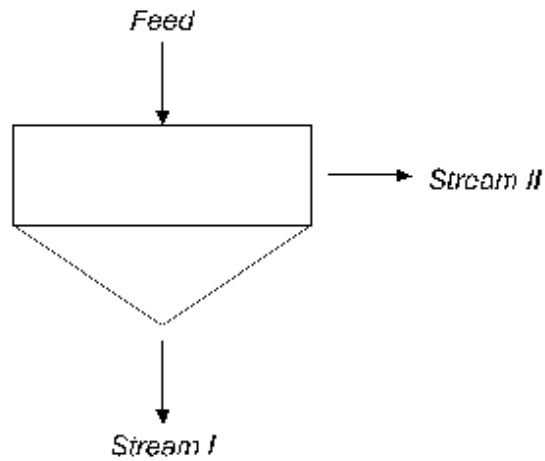
***Effects of Sizing  
Equipment Efficiency  
on  
Product Gradation and Weight Splits  
by  
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**1. The Issue**

Sizing devices used commercially are unfortunately not 100% efficient. The splits that can be obtained are usually inferior to the results obtained in the laboratory. Often the capital and operating costs of equipment capable of high sizing efficiency cannot be justified by the economics or requirements of the process. To reconcile the desired sizing efficiency needed to obtain optimum process requirements with the sizing efficiency of equipment economically practical, it is often necessary or desirable to predict the gradation and weight splits of the products obtained based on the expectant efficiency of the contemplated sizing device or method. The less efficient the sizing device one must employ, the more important it is to know the results that may be expected for the establishment of the optimum flow sheet and the more difficult it becomes to estimate the results accurately.

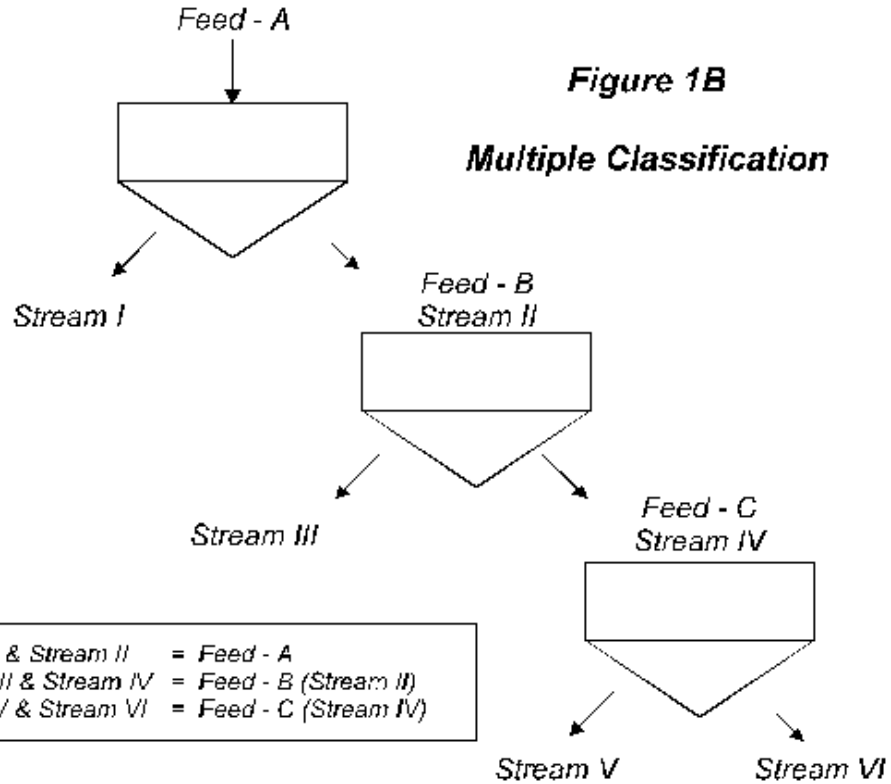
Air classification equipment which sizes materials by opposing the drag force created by an air stream acting on a particle with a combination of gravitational, inertial or centrifugal forces is substantially less efficient than, for example, a laboratory sieve analysis. Air classifiers allow oversized material to contaminate the fine fraction and leave undersized material in the coarse fraction. The techniques employed to predict the gradation and weight splits obtained from air classification outlined below are adaptable to other sizing devices less than 100% efficient such as screens, sifters and wet classifiers.

**Figure 1A**



**Figure 1B**

**Multiple Classification**



## 2. Efficiency

### 2.1 Sizing Efficiency Terminology

There is no recognized standard, industrial or otherwise, to express the efficiency of a sizing operation when a size overlap between the fractions obtained occurs, nor is there agreement in the terminology employed which varies from process to process and industry to industry. To avoid possible confusion or misunderstanding, the following definitions are used throughout this paper:

<b>Feed</b>	The particle size distribution of the material which is split in two separate fractions. Normally this is the material being discharged from a mill or other process piece of equipment and fed to a classifier. In a classifier producing more than two sized fractions, the feed is the size analysis of the material stream from which two separate material streams are created as shown in Figure 1. The feed is equal to 100% or any weight unit.
<b>Coarse Fraction</b>	The coarser fraction of the two fractions obtained from the split of the feed. In milling application, it is often referred to as reject. It is also mentioned as "Tailing," "Oversize," and in some process work as "Product." It is expressed as a percent of feed by weight.
<b>Fine Fraction</b>	The finer of the two fractions obtained from the split of the feed. It is sometimes referred to as "Product," "Dust," "Undersize." It is expressed as a percent of the feed by weight.
<b>Oversize</b>	The material found in the fine or coarse fraction and feed which is coarser than the stated cutpoint. It is expressed as a percent of the feed.
<b>Undersize</b>	The material found in the coarse or fine fraction and feed which is finer than the stated cutpoint. Expressed as a percent of the feed.
<b>Cutpoint</b>	The reference size employed to indicate product specification. It is normally referred to as a specific mesh size or micron size.
<b>Theoretical</b>	The size particle which has equal probability of being found in the coarse or fine fraction.
<b>Product</b>	The fine or coarse fraction meeting specification.

Figure 2 illustrates the above definitions with the exception of the Theoretical Cutpoint, which is illustrated later on in the text.

**Figure 2**

<b>Feed Analysis</b>					
	+ 60	Mesh	=	17.0	
- 60	+ 100	Mesh	=	17.5	<b>Oversize</b>
- 100	+ 140	Mesh	=	10%	<b>57.5% +200 mesh</b>
- 140	+ 200	Mesh	=	13%	
					<b>Cutpoint</b>
- 200	+ 325	Mesh	=	15	
- 325	+ 30	μ	=	10%	<b>Undersize</b>
- 30 μ	+ 20	μ	=	7.5%	<b>42.5% -200 mesh</b>
- 20 μ			=	10%	

<b>200 Mesh Classification</b>	
Fine Fraction 30% of feed	Coarse Fraction 70% of feed
Fines = 90% - 200 mesh	Coarse 78% +200 mesh
Oversize = 10% +200 mesh x 30% = 3%	Oversize = 78% +200 mesh x 70% = 54.5%
Undersize = 90% -200 mesh x 30% = 27%	Undersize = 22% -200 mesh x 70% = 15.5%
If Product Specification = 90% - 200 mesh	
Product = Fine Fraction	

## 2.2 Overall Efficiency

To determine classifier performance from a given or assumed feed, it is necessary to know the sizing efficiency of the device employed. There are several recognized methods of determining overall sizing efficiency.

The most commonly employed formula expresses sizing efficiency as the ratio between the undersize in the fine fraction and the undersize in the feed.

The reverse can also be employed when the emphasis is on the coarse product. The efficiency is then the ratio between the oversize found in the coarse fraction and the oversize in the feed. However, the efficiency ratio or percentage is no longer the same as it was calculated when using the undersize fractions in the fines and feed.

There is another deficiency in the formula. When a sample is split in such a way that the fine and coarse fraction are identical, in other words no classification takes place, the classification efficiency calculated by the above formula is 0.5 or 50%. This is a better starting point than 0% when you are selling classification equipment but it gives an erroneous impression of equipment overall efficiency.

A more sophisticated formula eliminates these deficiencies. In this formula, the sizing efficiency is identical to the one stated above, but a correction factor is introduced by subtracting from the calculated efficiency ratio, the ratio of oversize found in the fine fraction to the oversize available in the feed.

This formula is still far from perfect, as it makes the erroneous assumption that the oversize found in the fines is not product. When a product is made in a milling operation to a specification of perhaps 90% minus 200 mesh, the 10% plus 200 mesh is still product that must be extracted from the feed. The above formula penalizes the classifier efficiency; however, by subtracting a fraction of the fines oversize from the overall recovery. This results in making the equipment appear less efficient. You will not see this formula employed by classifier salesmen.

Actually, the true efficiency of a classifier when extracting a product expressed as a percent of a stated size, when the product extracted meets the specification, is the ratio of the product expressed as a percentage of feed to the amount of product available in the feed.

The following example illustrates the workings of the three formula just mentioned (see Figure 3a and 3b).

Figure 3A

First Formula      Efficiency =  $\frac{\text{Undersize}_{in\_Fines}}{\text{Undersize}_{in\_feed}}$  or  $\frac{\text{Oversize}_{in\_Coarse}}{\text{Oversize}_{in\_Feed}}$

Using Data from Figure 2      Efficiency =  $\frac{27}{42.5} = 0.635$  or  $\frac{54.5}{57.5} = 0.945$

Second Formula      Efficiency =  $\frac{\text{Undersize}_{in\_Fines}}{\text{Undersize}_{in\_Feed}} - \frac{\text{Oversize}_{in\_Coarse}}{\text{Oversize}_{in\_Feed}}$

Or

Efficiency =  $\frac{\text{Oversize}_{in\_Coarse}}{\text{Oversize}_{in\_Feed}} - \frac{\text{Undersize}_{in\_Coarse}}{\text{Undersize}_{in\_Feed}}$

Using Data from Figure 2      Efficiency =  $\frac{27}{42.5} - \frac{3}{57.5} = 0.58$

Or

Efficiency =  $\frac{54.5}{57.5} - \frac{15.5}{42.5} = 0.58$

Figure 3B

Third Formula      Efficiency =  $\frac{\text{Product}_{Obtained\_from\_Feed}}{\text{Product}_{Available\_in\_Feed}}$

Using Data from Figure 2      Efficiency =  $\frac{30}{42.5/0.90} = 0.635$

### 2.3 Overall Efficiency Not Useful

The problems encountered in predicting sizing performance from overall efficiency are analogous to describing a curve with just one point. The percentage or ratio efficiency figure calculated for an actual installation is limited to describing the effect of the basic efficiency of the device employed on one specific feed when making one specific product identified in one specific manner. Since you can change the overall efficiency by some thirty percentage points for any specific classification by just changing the reference size identification of the product, one has the idea of the relative uselessness of overall efficiency as a performance prediction tool

### 2.4 Fractional Efficiency Determination

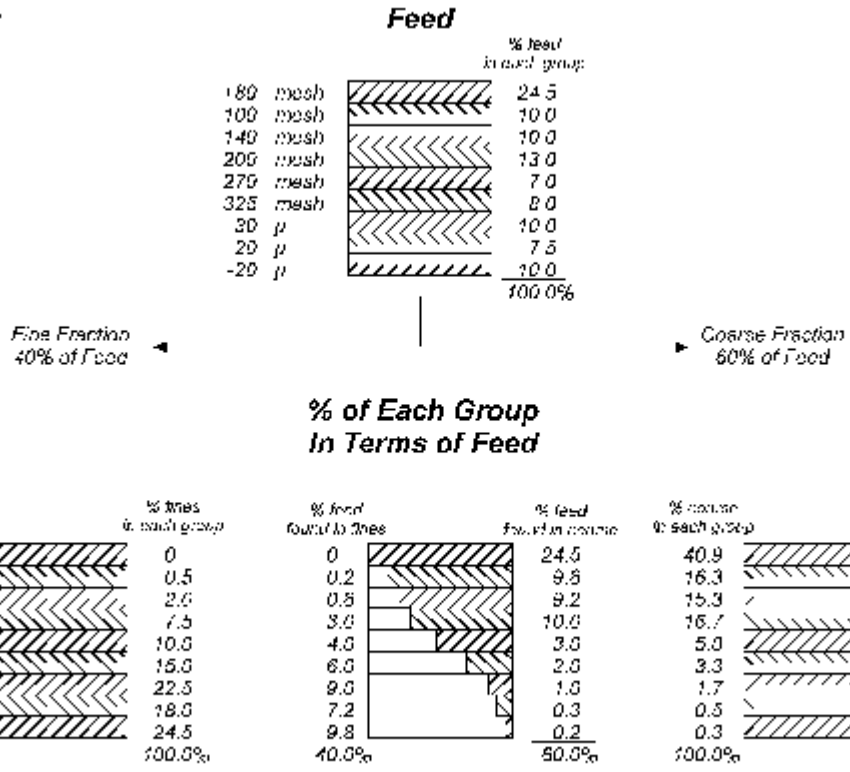
In order to obtain a worthwhile yardstick with which to measure the efficiency of a sizing device which will remain true from one material to another, regardless of cutpoint, product specification, feed gradation, etc., it is necessary to study the efficiency of the device, not at one cutpoint, but at a whole series of cutpoints covering the complete size range of the material being classified. In other words, obtaining the fractional efficiency of the device for specifically identified size groups. The smaller the size group, the better - up to a point which is dictated by the size analysis method employed and its reproducibility. Normally, the standard size nest of screens employed in sieve analysis, and increments of 10 microns for sub-mesh analysis, form satisfactory size groups as shown in Figure 4 below.

The efficiency for each size group is the ratio of extracted material found in the fine or coarse fraction to the material available in the feed. Since the fine and coarse fraction analyses do not always match mathematically the feed analysis, it is better to use a mathematically reconstructed feed from the fine and coarse fraction than the actual feed analysis. In the event the weight split is unknown, it must be calculated from the size analysis of the fine, coarse and feed fractions. This is a trial-and-error problem, as in this work the sum of the parts never equals the whole. The technique is once again to break the analysis of the fine, coarse and feed fraction into groups and only select the groups which have a substantial percentage of the material for both the coarse and fine fractions. The split for each group is readily found by simple Algebra (see Figure 5).

The percentage split calculated for each group might vary by as much as 10%. An average split then, must be judiciously selected. The reconstructed feed analysis will be at a slight variance with the actual feed analysis. Close scrutiny; however, will indicate that any gain or loss in each size group is made up in the adjacent group.

Once a mathematically correct distribution is obtained, the percentage of the feed found in the fine or coarse fraction is calculated. Figure 4 shows the feed split calculations for each size group graphically represented. Figure 6 shows the actual distribution of the feed between the coarse and fine fraction in greater details with a curve drawn through the midpoint of each size group split. This curve is the Fractional Efficiency Curve.

**Figure 4**

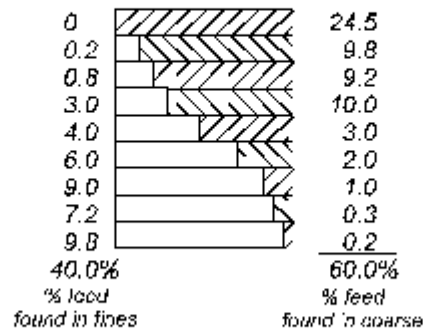


**Figure 5**

Given	A = % Fines in Selected Group B = % Coarse in Selected Group C = % Feed in Selected Group
If	X = % Fine Fraction Obtained From Total Feed Y = % Coarse Fraction Obtained From Total Feed
Also	AX + BY = 100C X + Y = 100 Y = 100 - X
Substituting	AX = 100B - BX = 100C
If	A = 10% B = 5% C = 7%  10X - 5X = 700 - 500 5X = 200 X = 40%

**Figure 6**

**% of Each Group In Terms of Feed**



## 2.5 Log Probability Plot

Note: Even with personal computers, software does not exist that allows Fractional Efficiency to be plotted effectively. Recommendation is that these continue to be plotted by hand.

The curve shown in Figure 6 is for only one specific classification. This curve will change shape, depending on the size group selection, spacing and cutpoint. In order to compare various fractional efficiency curves obtained from various classifications, some standardization method of reporting them must be established. For this purpose, logarithmic probability graph paper is very convenient. Log probability graph paper has normally a two-cycle log scale as the ordinate with the probability scale as the abscissa (Figure 7). This type graph paper, in the writer's opinion, should be always employed for graphic representation of material size analysis, as most particle size distributions plot close to a straight line and great visual emphasis is given to the extremes of the particle size distribution.

When the graph paper is used for plotting Fractional Efficiency Curves, the ordinate refers to particle size in microns - starting at 1, 10 or 100 microns, depending on the size distribution of the material classified. As sieve sizes have a corresponding micron size, the sieve sizes can be spotted on the ordinate at the equivalent micron size. The space between each sieve size becomes the group for which the fractional efficiency is determined. The abscissa refers to the percentage of the original feed found in the fine or coarse fraction.

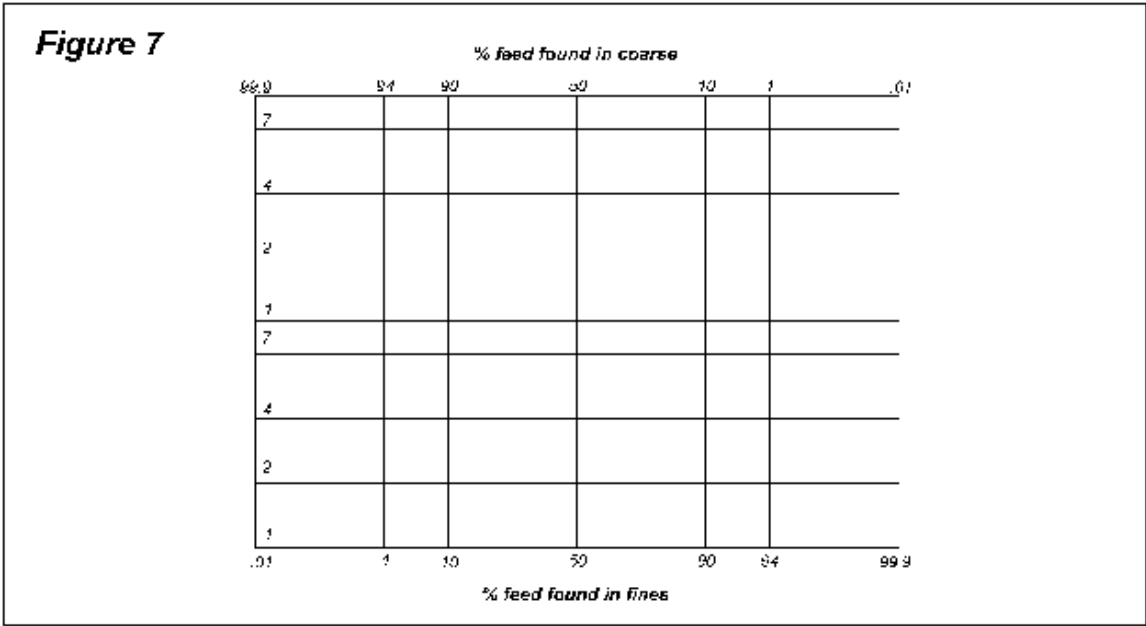
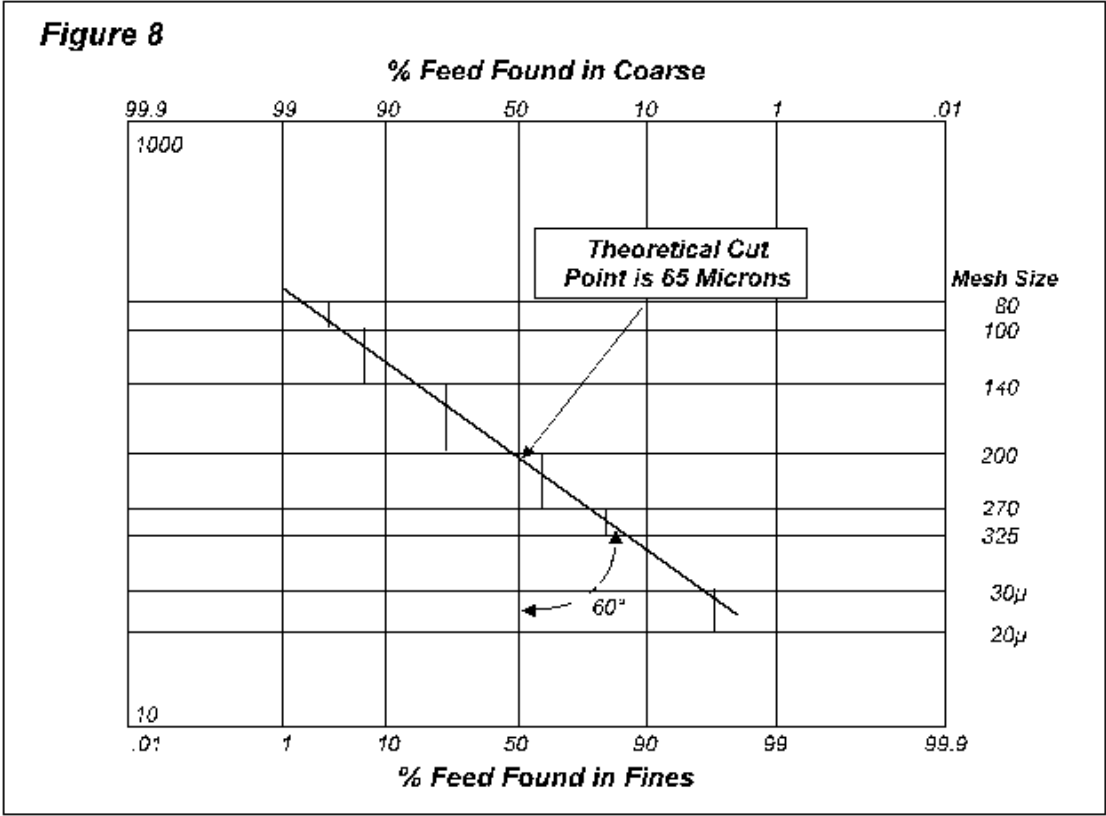


Figure 8 shows the fractional efficiency curve shown previously in Figure 6 replotted on log probability paper. Three things should be noted. One is that the fractional efficiency curve is a reasonably straight line. Two - the angle formed between the fractional efficiency curve and the ordinate is a direct index of equipment efficiency. A 90 angle, or a fractional efficiency line perpendicular to the ordinate, indicates 100% efficiency of the sizing device. 0 or a fractional efficiency line parallel to the ordinate equals 0 efficiency or no classification. This line, by the way, must overlay the 50-50 probability line. Three - the point at which the fractional efficiency line crosses the 50-50 probability line indicates the particle size which has equal opportunity for migrating to the coarse or fine fraction. This is the theoretical cutpoint. It is a reference point to locate the fractional efficiency curve position on the graph.



## **2.6 Practical Aspects of the Fractional Efficiency Plot**

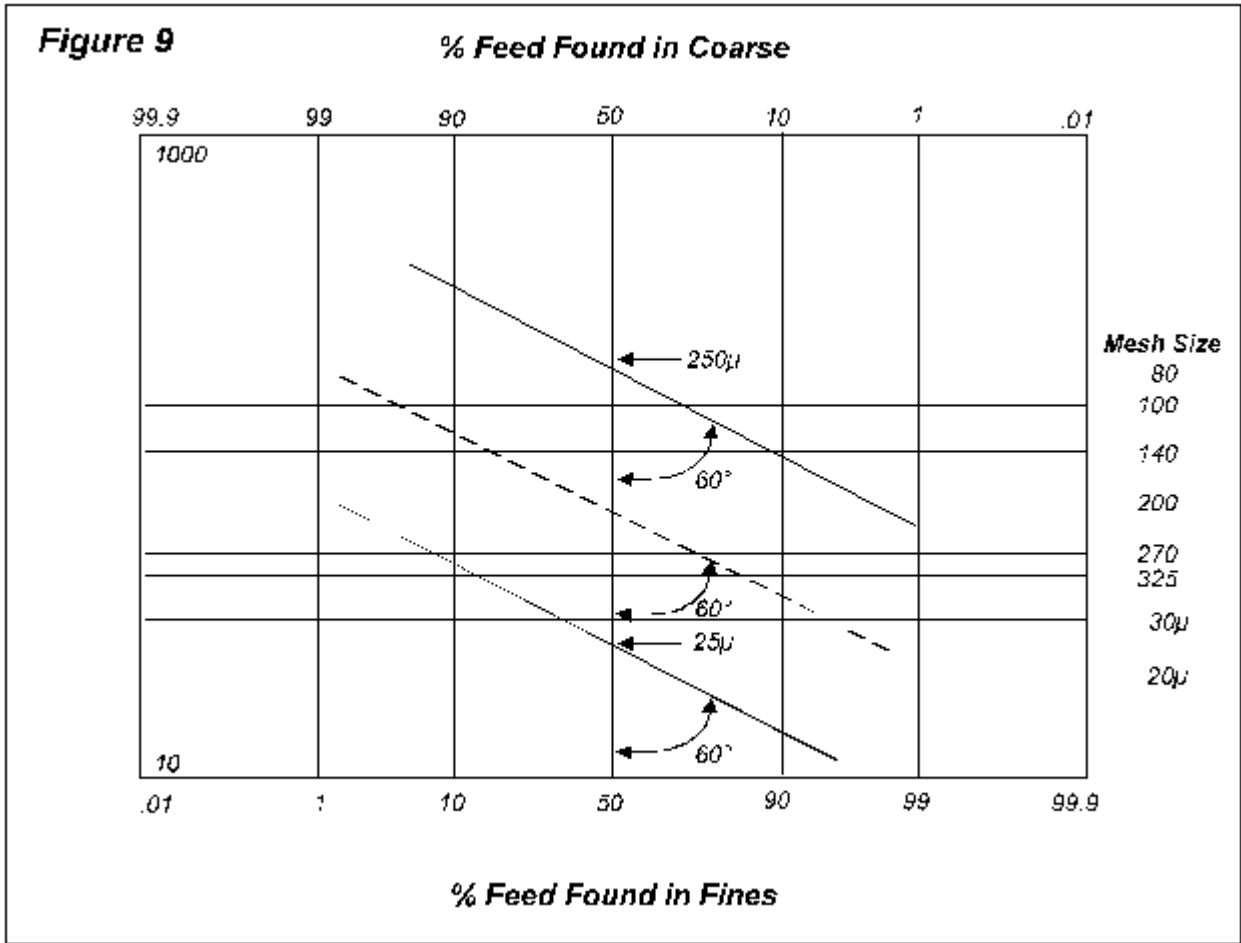
The theoretical cutpoint and the angle are all that is required to describe all aspects of a classification. With these two items assumed or obtained, it is simple mathematics to calculate the weight split and complete size analysis of the fine or coarse fraction from a given feed analysis by just reversing the calculation process shown above for the determination of a fractional efficiency curve. If the product must meet a specific gradation, the feed gradation required to meet product specification can be determined. If the sizing efficiency of the equipment is fixed, or of a limited magnitude, the calculated feed gradation requirement in turn helps in the selection of the equipment employed to produce the feed, be it a mill, crystalizer, pelletizer or other piece of process equipment.

## **2.7 Air Classifier Fractional Efficiency Characteristics**

Efficient air classifiers, operating on materials having particles with close correlation between their physical size as measured by sieves and their settling velocities in air, will have essentially straight fractional efficiency curves. The angle can vary between 50 and 75. An angle of 60 is most usual and can be assumed if pilot plant data or commercial installation data is not available.

If the classifier is operated within a stable operating range, the fractional efficiency curve will not change at various cutpoints (Figure 9). This is a very useful condition when pilot plant data needs to be extrapolated to new cutpoints or new feed gradations.

The upper and lower sections of the curve passing the 5-10% mark are the sections most likely to deviate from a straight line. There is a limit to the extraction of extreme fines from the feed, regardless of the efficiency of the air classifier. It rarely exceeds 98% for particles below 200 mesh. When the equipment is operated at high cutpoints, it is possible to extract a greater percentage of particles between 200 and 100 mesh than particles below 325 mesh. This phenomenon is caused mostly by the adhering properties of the extreme fines to the coarse particles



If the collection system on the air entrained fines is deficient and the air is returned to the classifier, it can be expected that at least 40% of the extreme fines lost by the collector will end up in the oversize fraction through reinjection, regardless of the classifier efficiency.

The maximum size particle to be found in the fine fraction is also hard to predict with accuracy, as there is always the possibility that the feed contains odd-shaped particles with sufficient deviation from the norm which end up in the fine fraction in sufficient number to deviate the fractional efficiency curve from a straight line.

## 2.8 Limitations

If the material to be classified has unusual physical characteristics, the fractional efficiency curve will deviate from the straight line. Flyash, for example, which sometimes contains large particles in forms of bubbles or sections of one, can develop weird S-shaped fractional efficiency curves. This is due to the difference in sizing principles between air classifications, which looks at a particle on a weight and surface area basis, and the sieve analysis which only looks at particles in a two-dimensional plane.

Obviously, the less correlation there is between the commercial and laboratory evaluation sizing methods employed, the greater the deviation from a straight line fractional efficiency curve. When this occurs, it is difficult to use the F.E. curve obtained from one installation for estimating results in another operation, when there is a change in cutpoint or size analysis.